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ANALYTICAL MODEL FOR CONTROLLING A MARTENSITIC PHASE UNDER LOADING

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A model for calculating the amount of martensitic phase is proposed based on the assumption that the derivative of this amount with respect to temperature is proportional to the product of the amounts of the martensitic and austenitic phases. The approach developed in the paper allows a reliable description of the deformation behavior of materials during thermal cycling in intervals of martensitic transitions, including incomplete phase transformations.

Key words: *martensitic and austenitic phases, phenomenological approach, thermal cycling, model for calculating the amount of martensitic phase.*

Calculating stresses and strains in materials with martensitic inelasticity is a difficult problem. Existing models for calculating the properties of materials with phase transformations allow a solution of this problem only in particular cases. In addition, the question of the choice of the structural parameters specifying the state of material remains open. A mathematical description of martensitic inelasticity can be performed using the structurally analytical theory of strength [1]. However, using this theory to solve concrete engineering problems is greatly complicated even in a two-level formulation. There are examples of solutions of such problems in a single-level formulation [2], in which it is assumed that the derivative of the phase strain deviator with respect to the amount of the martensitic phase q is equal to the product of a linear combination of the stress and strain deviators and the amount of the austenitic phase $(1 - q)^n$ (n is a material parameter). We note that, in the approach described above, the main controlling parameter is the amount of the martensitic phase, which, in turn, is an ambiguous function of the temperature T.

In the present paper, a single-level phenomenological approach is proposed which allows one to formulate problems of martensitic inelasticity during thermal cycling of a material under loading in terms of engineering mechanics. The phenomenological model is based on the following hypotheses and assumptions consistent with experimental facts and proposed earlier in [3].

1. In the proposed model, a martensitic phase is controlled by varying the temperature and stress. It is assumed that the presence of a martensitic phase during thermal cycling in martensitic transition intervals is determined by a temperature hysteresis taking into account the Clausius–Clapeyron relation [1].

2. It is assumed that the increment of the strain responses $d\varepsilon_{ij}$ is proportional to the product of the increment of the phase $d\Phi$ and the factor $f(\sigma_{ij})$, which, according to experimental data, for low stresses (0.1–0.2 of the material strength) is proportional to σ_{ij}.

As is known, the amount of a martensitic phase varies reversibly during heat changes in martensitic transition intervals. The region of phase changes is bounded by a closed curve $M_{\rm i}M_{\rm f}A_{\rm i}A_{\rm f}$, where $M_{\rm i}$, $M_{\rm f}$, $A_{\rm i}$, and $A_{\rm f}$ are the characteristic phase-transition temperatures (Fig. 1). The variables Φ and T are the amount of the martensitic phase and temperature, respectively. A decrease in the temperature leads to an increase in the fraction of the martensite (this process is considered a direct one), and an increase in the temperature leads to the reverse process — transition of the martensite to austenite phase.

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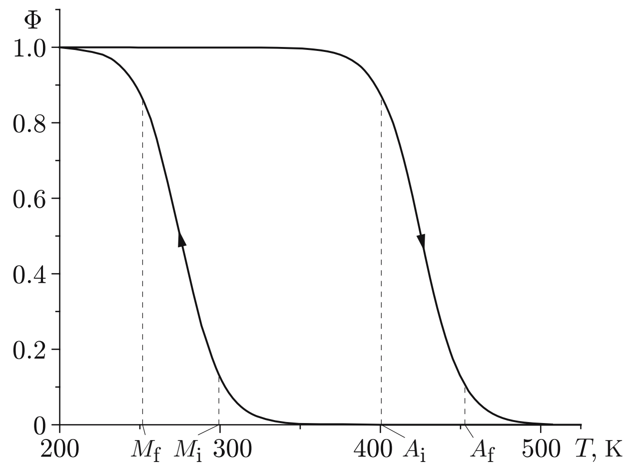

Fig. 1. Amount of a martensitic phase versus temperature during thermal cycling of the material in martensitic transition intervals.

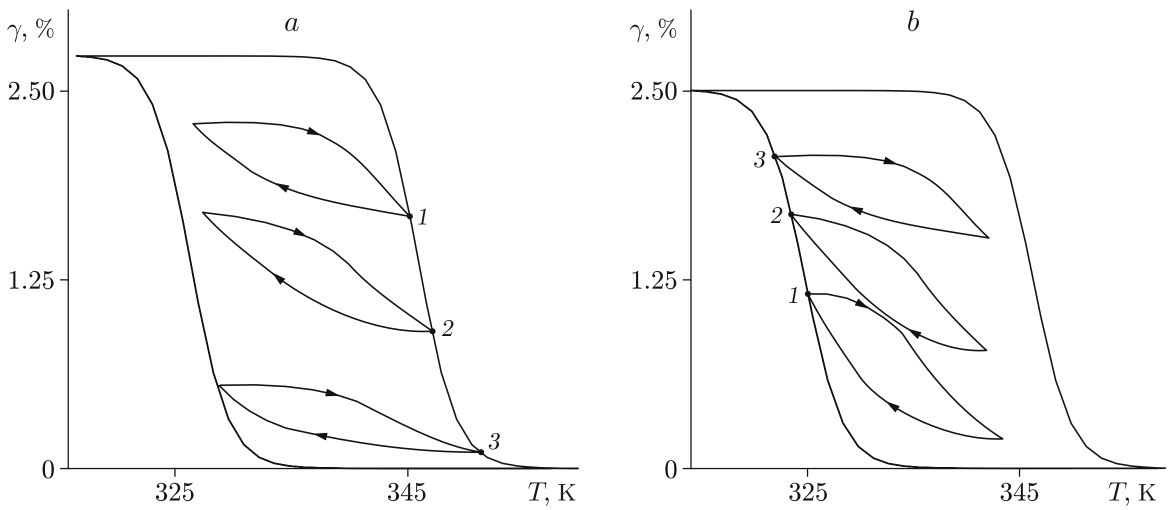

Fig. 2. Experimental curves of shear strain versus temperature for an 50/50 Ti/Ni alloy in alternating cooling and heating cycles under a stress $\tau = 75$ MPa in an incomplete martensitic transformation [4]: (a) transformation of martensite to austenite; (b) transformation of austenite to martensite; points 1, 2, and 3 show the beginning of thermal cycles.

We note that, as $\Phi \to 0$ and $\Phi \to 1$, $\Phi'(T) \to 0$. We choose the function $\Phi(T)$ from the condition $\Phi'(T) = -K\Phi(1-\Phi)$, where K is a positive constant which defines the slope of the curve. The value of K is chosen so that, at the points $M_0 = (M_\mathrm{i} + M_\mathrm{f})/2$ and $A_0 = (A_\mathrm{i} + A_\mathrm{f})/2$, the values of $\Phi(T)$ are equal to 0.5. As a result, we have $K_\mathrm{m} = 4/(M_\mathrm{i} - M_\mathrm{f})$ and $K_\mathrm{a} = 4/(A_\mathrm{f} - A_\mathrm{i})$, where K_m and K_a are the values K for the ascending and descending portions of the curve $\Phi(T)$, respectively. The equation

$$\Phi(T) = H(-\dot{T})\Big[1 + \exp\left(K_\mathrm{m}(T - M_0)\right)\Big]^{-1} + H(\dot{T})\Big[1 + \exp\left(K_\mathrm{a}(T - A_0)\right)\Big]^{-1}$$

describes the dependence of the amount of the martensitic phase Φ on the temperature T during complete thermal cycling in the characteristic temperature intervals. Here $H(x)$ is a Heaviside function [$H(x) = 1$ at $x \geqslant 0$ and $H(x) = 0$ at $x < 0$] and $\dot{T}$ is the time derivative of the temperature.

Figure 2 shows experimental temperature dependences of the shear strain for a 50/50 Ti/Ni alloy in alternating cooling and heating cycles under a stress τ in an incomplete interval of martensitic transformation. The temperature dependence of the axial strain is nearly the same [4, 5].

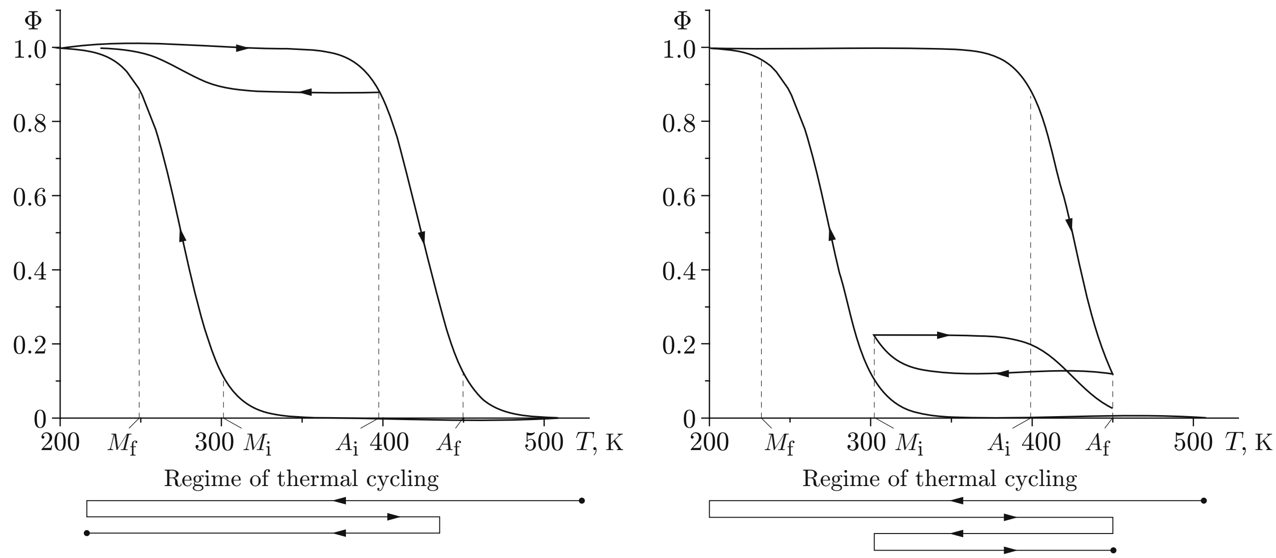

Fig. 3. Calculated temperature dependences of the amount of the phase in an incomplete interval of martensitic transitions for the model material ($M_i = 300$ K, $M_f = 250$ K, $A_i = 400$ K, and $A_f = 450$ K) in two regimes of thermal cycling.

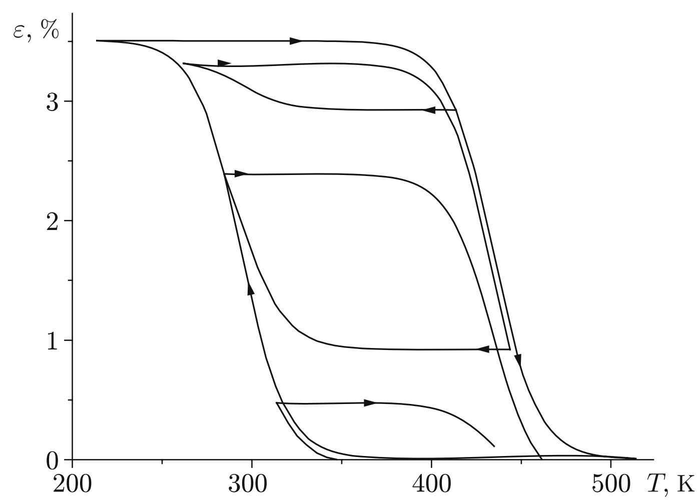

Fig. 4. Calculated curve of strain versus temperature for the model material during alternating cooling and heating under a stress τ in incomplete martensitic transformation.

In the proposed kinematic model for calculating the amount of the martensitic phase for incomplete phase transformations, the following analytical dependence is adopted:

$$\Phi(T) = H(-\dot{T})\Big\{\Phi_1 + (1 - \Phi_1)[1 + \exp(K_{\rm m}(T - M_0))]^{-1}\Big\} + H(\dot{T})\Phi_1[1 + \exp(K_{\rm a}(T - A_0))]^{-1}.$$

The model assumes that the curve $\Phi(T)$ through the point (T_1, Φ_1) is obtained from the hysteresis curve in the complete interval of martensitic transformation (see Fig. 1) by compression by a factor of $1 - \Phi_1$ and an upward shift by Φ_1 with decreasing T and by compression by a factor of Φ_1 time with increasing T (Φ_1 is the amount of the initial phase; T_1 and T_2 are the initial and final temperatures, respectively, in the interval of the martensitic transition). Calculated temperature dependences of the amount of the phase in complete and incomplete martensitic transitions are shown in Fig. 3.

The variation in the amount of the martensitic phase Φ during thermal cycling under loading is described by the following analytical relation:

$$\Phi(T) = H(-\dot{T})\Big\{\Phi_1 + (1-\Phi_1)[1+\exp(K_{\mathrm{m}}(T^* - M_0))]^{-1}\} + H(\dot{T})\Phi_1[1+\exp(K_{\mathrm{a}}(T^* - A_0))]^{-1}.$$

Here $T^* = T - K_{ij}\sigma_{ij}$ is the effective temperature determined from the Clausius–Clapeyron relation, σ_{ij} is the stress tensor, and K_{ij} are the components of the second-rank tensor. In the simplest formulation, it is assumed that $K_{ij} = \mathrm{const}$ ($K_{ij} = 0$ for $i = j$ and $K_{ij} = \alpha$ for $i \neq j$) [6].

According to experimental data on the effect of mechanical tensile stress on the characteristic temperatures of martensitic transitions for titanium nickelide of nearly equiatomic composition, $\alpha \approx 0.14\,\mathrm{K}\cdot\mathrm{MPa}^{-1}$ [7, 8]. Figure 4 gives a curve of strain versus temperature for the model material under alternating cooling and heating under a stress τ in incomplete martensitic transformation.

The strain increment $d\varepsilon_{ij}$ is proportional to the increment of the amount of the phase and load σ_{ij}: $d\varepsilon_{ij} = a_i\sigma_{ij}\,d\Phi$. In the calculations, the coefficient a_i — the scalar parameter corresponding to the strain capacity during thermal cycling of the loaded material in martensitic transition intervals — is set equal to 10^{-10} Pa^{-1}. Since the strains due to shape memory and plasticity effects during direct transformation are practically equal, one can set $a_1 = a_2 = a$.

Thus, the proposed model for calculating the amount of martensitic phases provides a fairly accurate description of the deformation behavior of material during thermal cycling in martensitic transitions, including incomplete phase transformations.

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